

Abstract

Kan complexes play a central role in simplicial homotopy theory, being the fibrant objects with respect to the standard model structure.

A few years ago, Valentin Shehtman associated to each first-order theory certain simplicial set whose n -simplices are given by the n -types of that theory. For the purposes of his investigation, it is important that this simplicial set satisfies the so-called Beck–Chevalley condition: for any $i < j$ and any two simplices a, b such that the i th face of a coincides with the $(j - 1)$ st face of b , there exists a simplex x whose j th and i th faces are a and b , respectively.

It is easy to see that every Kan complex satisfies the Beck–Chevalley condition, but whether the converse is true is not yet known.

We study the Beck–Chevalley condition for those simplicial sets that arise as the Duskin nerves of small 2-categories. In particular, we investigate what kind of conditions a monoid must satisfy in order for the Duskin nerves of some naturally defined sub-2-categories of the 2-category of endofunctors and their natural transformations of M viewed as a category with a single object to satisfy the Beck–Chevalley condition.