Some methods of complex analysis in the theory of functions of a real variable

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The paper is of a referential nature and explores the role of complex analysis methods in addressing significant problems in the theory of real variable functions. The first part highlights the application of the Riemann zeta function in the study of the asymptotic distribution of prime numbers. It emphasizes the fundamental contributions of Euler, Legendre, Chebyshev, Hadamard, and de la Vallée-Poussin to the proof of the Prime Number Theorem, which is grounded in the tools of complex analysis. The holomorphic and meromorphic properties of the zeta function are examined in detail, as they form the analytical basis for obtaining asymptotic estimates related to the distribution of primes.

The second part of the paper presents a problem concerning the enumeration of subsets of a given set whose elements sum to a value divisible by a fixed integer. The solution is based on the use of the roots of a complex polynomial. Through analytical methods, an explicit formula is derived that expresses the number of such subsets.

Overall, the paper clearly demonstrates the theoretical depth and versatility of complex analysis methods, both in real analysis and number theory.

References

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